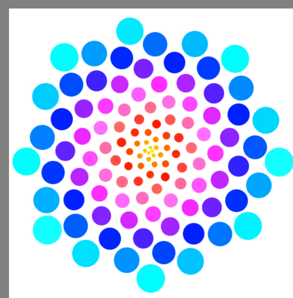
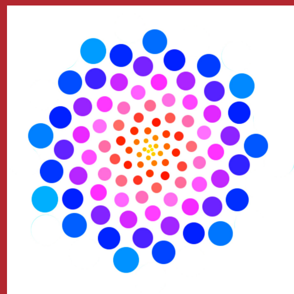
SAT TIER 1 / MODULE I:

Mathematics



# NUMBERS AND OPERATIONS

MODULE ONE



# COUNTING AND PROBABILITY

## Before You Begin

When preparing for the SAT at this level, it is important to be aware of the "big picture" problems you will need to be able to solve. Therefore, at this stage we will focus more on the task of identifying the kinds of problems you should be familiar with, and techniques for solving these problems. Later, as you develop your assessment skills, you will then be in a place to deal with more specialized problems of greater difficulty.

## 1 ▪ Probability

There is a very important point that we need to cover before we tackle probability. There is only one probability question on every SAT. That means you will encounter a probability question, but it also means that you will not encounter more than one. If probability is difficult for you, this fact should be good news. The probability question might be an easy one, in which case you should rejoice since encountering a difficult topic in an easy way will allow you to sidestep a potentially difficult problem. Alternatively, the probability question might be hard. This case, too, is good news unless you are aiming for a score in the mid- to high-700s. A hard probability question is good news because it means that the problem is essentially killing two birds with one stone: a hard question involving a hard topic means that you don't necessarily have to worry about that question. With that said, let us look at probability.

Figure 1.1: Here is caption.

Table 1.1: Here is caption.

### 1.1 Geometric Probability

We begin with geometric probability. The test might ask a question that gives you two shapes and asks what the probability is that a random point is in one of the two shapes. Let's look at a couple of examples to understand the way a question might appear.

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The first question deals with area and the second with length. In both cases, the solution is essentially the same.

**Instructor Note:**

A slight qualification is in order. The SAT might actually use the exclamation point, but if it does so, it will define the term. In other words, factorials fall in the same category as concepts such as perfect numbers, arithmetic. . .

To see these rules in action, we'll solve the problems below.

**Important Note:**

Area Questions: The probability that the point is in the smaller shape is $\frac{\text{area of smaller shape}}{\text{area of larger shape}}$.

Length Questions: The probability that the point is on the shorter length is $\frac{\text{shorter length}}{\text{larger length}}$.

## Calculator Tip:

It is worth learning how to calculate ${}_nC_r$ on your calculator. Different models have different approaches. For example, on a TI-83, to calculate the number of ways of picking three things out of seven, you enter 7 ${}_nC_r$ 3. On a TI-89, however, the same problem should be entered as ${}_nC_r$ (7, 3). Make sure you know your technology!

To see these questions in action, we'll solve the problems below.

### *EXAMPLE 1*

How many ways can three students be picked from a group of five to serve as class officers if order does not matter?

***Solution***

We can use the formula or use ${}_5C_3$ on a calculator. Using the formula, we get $\frac{5!}{3!2!} = 10$.

Example without a solution:

### *EXAMPLE 2*

Jeanette plans to take one science course, one language course, one sociology course, and one English course for her sophomore year at college. If there are 4 available science courses, 5 available language courses, 8 available sociology courses, and 3 available English courses, how many different schedules are possible? (Assume there are no time conflicts between any two courses.)

Setting the example number, example with questions and answers:

### *EXAMPLE 22*

From a group of twelve students, a president and vice president will be selected.

**1.** How many different pairs of these officers are possible?

(A) 2

(B) 4
(C) 9
(D) 0
(E) 132

**Solution**

If we pick the president first, there are 12 possible students. Once we've chosen the president, any of the remaining 11 students can be vice president. Thus, the total number of possible pairs is $12 \cdot 11 = 132$.

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**EXAMPLE 23**

Helena and Isabella are in a class of 12 students, and two students from that class will be chosen at random to be president and vice president. What is the probability that Helena will be president and Isabella will be vice president?

**Solution**

The fast answer is to note that there are 132 possible pairs (from the previous example), and the Helena-Isabella tandem is one of those pairs. Thus, the answer is $\frac{1}{132}$.

However, we can also approach the problem this way: The probability that Helena will be chosen to be the president is $\frac{1}{12}$, and, assuming that Helena is indeed chosen to be president, the probability that Isabella is chosen as vice president is $\frac{1}{11}$. Thus, the probability that Helena is president and Isabella is vice president is $\frac{1}{12} \cdot \frac{1}{11} = \frac{1}{132}$

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You might have noticed that this method is not as mathematical as you might expect. In other words, we are not using an equation or formula here. There are formulas for this type of problem, but the SAT will never explicitly require them. That said, we will give a formula for solving problems in which one group is selected from a larger group.

If order does not matter, the number of ways of picking $r$ things out of a group of $n$ things (where $n \sim r$) is $\frac{n!}{r!(n-r)!}$.

The formal symbolism for this formula is $\binom{n}{r}$ or ${}_nC_r$, but you will never see that symbolism on the actual SAT.

Three important points:

- You will not be asked problems that require large numbers. So you'll never be asked, for example, how many ways there are to select 6 people out of 10. (The answer is 210, which is far too many to list.)
- You should not rely on being able to use $_nC_r$ on your calculator since it is very easy to create a problem that will not allow you to use this function.
- Do not worry about how you will tell whether order matters. The test will be clear.

## Check for Understanding

In this lesson, we've tried to instill in you methods based more on thinking and understanding than on rigid application of formulas. Formulas are helpful, of course, but they al so have their limits, and the SAT often pushes those limits. Moreover, the skills you gain if you learn these methods, particularly the ability to list options systematically, can help you in problems that are related even though they may at first not appear so. Consider the next example.

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***EXAMPLE 24***

Two people are to be chosen from a group of five- Daryl, Eddie, Francine, Georgia, and Hyun-to be representatives. If Daryl cannot be chosen with either Francine or Georgia, how many pairs are possible?

***Solution***

One way to do the problem is to list all 10 pairs of people. (We know there are 10 because we did this problem earlier with two concert tickets for five friends.) Those ten pairs are DE, DF, DG, DH, EF, EG, EH, FG, FH, and GH. Since two of these pairs violate the rules (DF and DG), they are eliminated, and there are 8 possible pairs.

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### *Challenge Problem*

Based on the setup in the previous example, what is the highest score that cannot be a total score? In how many different ways can a player get a total score of 25?

■ **Instructor Note:**

The answers to the two challenge questions are 6 and 3.

### *1.2 Combinations Without Order*

Suppose you have four friends you'd like to invite to a concert, but the problem is that you have only two extra tickets. How many possible pairs of friends can you take?

**1.** If an angle in a right triangle is chosen at random, what is the probability that its measure is less than 90°?

(A) $\frac{1}{3}$

(B) $\frac{1}{2}$

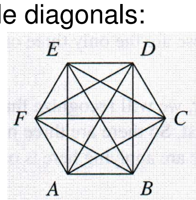
(C) $\frac{2}{3}$

(D) $\frac{3}{2}$

(E) It cannot be determined from the information given.

### Instructor Comment:

The term "diagonal" has been used on the real test to refer to a segment whose endpoints are non-adjacent vertices of a polygon. The figure below shows all possible diagonals:




$$A = \{1,2,3,4,5\}$$
$$B = \{3,4,5\}$$
$$C = \{1,2,6\}$$

**2.** If an integer is chosen at random from set A, what is the probability thatis in one of the other two sets?

(A) $\frac{1}{6}$

(B) $\frac{1}{5}$

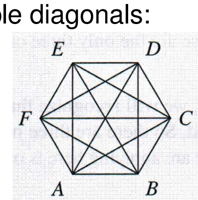
(C) $\frac{1}{3}$

(D) $\frac{1}{2}$

(E) 1

### Instructor Comment:

The term "diagonal" has been used on the real test to refer to a segment whose endpoints are non-adjacent vertices of a polygon. The figure below shows all possible diagonals:



$$1, 2, 3, 4, 5, 6, 7, 8$$

**3.** The value of $n$ is chosen at random from the list above. What is the probability that $n2 - 1$ is an odd number?

(A) $\frac{1}{8}$

(B) $\frac{1}{4}$

(C) $\frac{1}{2}$

(D) $\frac{5}{8}$

(E) $\frac{3}{4}$

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$$\begin{aligned} \text{Set A} &= \{1, 2, 3, 4, 5, 6\} \\ \text{Set B} &= \{2, 4, 6\} \end{aligned}$$

**4.** If an number is selected at random from set B, what is the probability that number is also in set A?

(A) 0

(B) $\frac{1}{6}$

(C) $\frac{1}{3}$

(D) $\frac{1}{2}$

(E) 1

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