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INTER-COMMODITY SPREADS AND IMPLIED PRICING

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Implied quoting is a characteristic feature of interest rate futures markets. Because these markets typically consist of a collection of very tightly connected products representing different points on a yield curve, spread products are an essential tool for effective trading. Spreads interact with the outright contracts to generate both hidden and visible implied liquidity, which can yield large improvements in both size and price for trades in the outrights.

CME intercommodity Treasury spreads involve a particularly complex set of implication relationships, because the ratios are typically not 1:1 as in other products such as Eurodollar futures. Proper understanding of the rules of implication and the ways that these prices interact can yield real improvements in trade execution.

The information in this report has been obtained by observing fills received by QB's execution algorithms, by carefully reading available documentation, by discussions with representatives of CME Group, and by direct testing in the CME certification environment.

INTERCOMMODITY SPREADS

An intercommodity spread (ICS) consists of two legs and a quantity for each leg. The spread itself and the two legs all have separate limit order books. Buying the spread results in going long the first leg and going short the second leg in the corresponding quantities, and selling the spread results in the the inverse position. The sizes assigned to each leg of some ICS may change between maturities in order to match the relative yield sensitivity between the two legs. For example, the TUL spread has front leg ZT (two year Treasury futures), and back leg UB (ultra Treasury bond futures). For December 2016, the TUL spread had a size ratio of 9:1, which changed to an 8:1 ratio for March 2017. Many product pairs have more than one active intercommodity spread, with different ratios, presumably to accomodate a variety of beliefs about the correct yield sensitivities.

GLOBEX SYMBOLS

We will use the Globex symbols to refer to the various contracts we consider, as in Table 1. In the rest of this note we will use the March 2017 FYT (ZF *vs.* ZN) spread in all illustrations and examples, both because it is typically the most liquid of the intercommodity spreads, and because its 3:2 quantity ratio makes it simple to remember which leg of the trade we are considering at any point.

IMPLICATION RULES

The quantity relationships may be captured by the three equivalant algebraic expressions

$$[\text{FYT}] = 3[\text{ZF}] - 2[\text{ZN}] \quad (1)$$
$$2[\text{ZN}] = 3[\text{ZF}] - [\text{FYT}] \quad (2)$$
$$3[\text{ZF}] = 2[\text{ZN}] + [\text{FYT}]. \quad (3)$$

The first of these (1) tells us that buying one lot of the FYT spread is equivalent to buying 3 lots of the ZF futures and selling 2 lots of ZN . Similarly, selling one lot of FYT is equivalent to selling 3 ZF and buying 2 ZN. In fact, a trade in FYT will settle by delivering the appropriate long and short positions in the underlying legs.

The second relationship (2) tells us that buying 3 lots of the ZF futures and selling one lot of the FYT spread is equivalent to buying 2 lots of the ZN futures. The FYT trade will deliver a short position in 3 lots of ZF, which will cancel the direct purchase of 3 lots. Similarly, selling 3 ZF and buying 1 FYT is equivalent to selling 2 ZN.

The six forms of implied trading for the FYT spread are shown in Figure 1, where we use negative signs to denote selling the corresponding contract. This ability to achieve the same trade in multiple ways provides additional liquidity that we can access in two ways:

- As described above, we can send simultaneous orders to the different components of the spread and outrights, called "legging." This can sometimes result in significant price improvement, as described in "Pricing Relationships" below. But it incurs "legging risk:" there is a chance only one part of the trade is completed before the price changes. In addition, legging the trade may incur additional commission fees to the exchange.
- Alternatively, in certain circumstances, the CME Globex matching engine will automatically fill our order using the implied liquidity from other markets. This guarantees a transaction free from legging risk, but for reasons discussed below in "Matching and Fills," will only result in additional liquidity, and not price improvement (barring serious dislocations between the markets).

PRICING RELATIONSHIPS

Prices on intercommodity spreads are listed based on the change in price in the outrights from the previous day's settlement price. Going long in the first leg of a spread at price A, and going short the second leg at price B (in appropriate sizes as shown in Figure 1), is equivalent to buying the spread at price

$$S = A - \bar{A} - \frac{B - \bar{B}}{r}, \quad (4)$$

where $\bar{A}$ and $\bar{B}$ are the previous day's settlement prices, and r is the price ratio from Table 1.

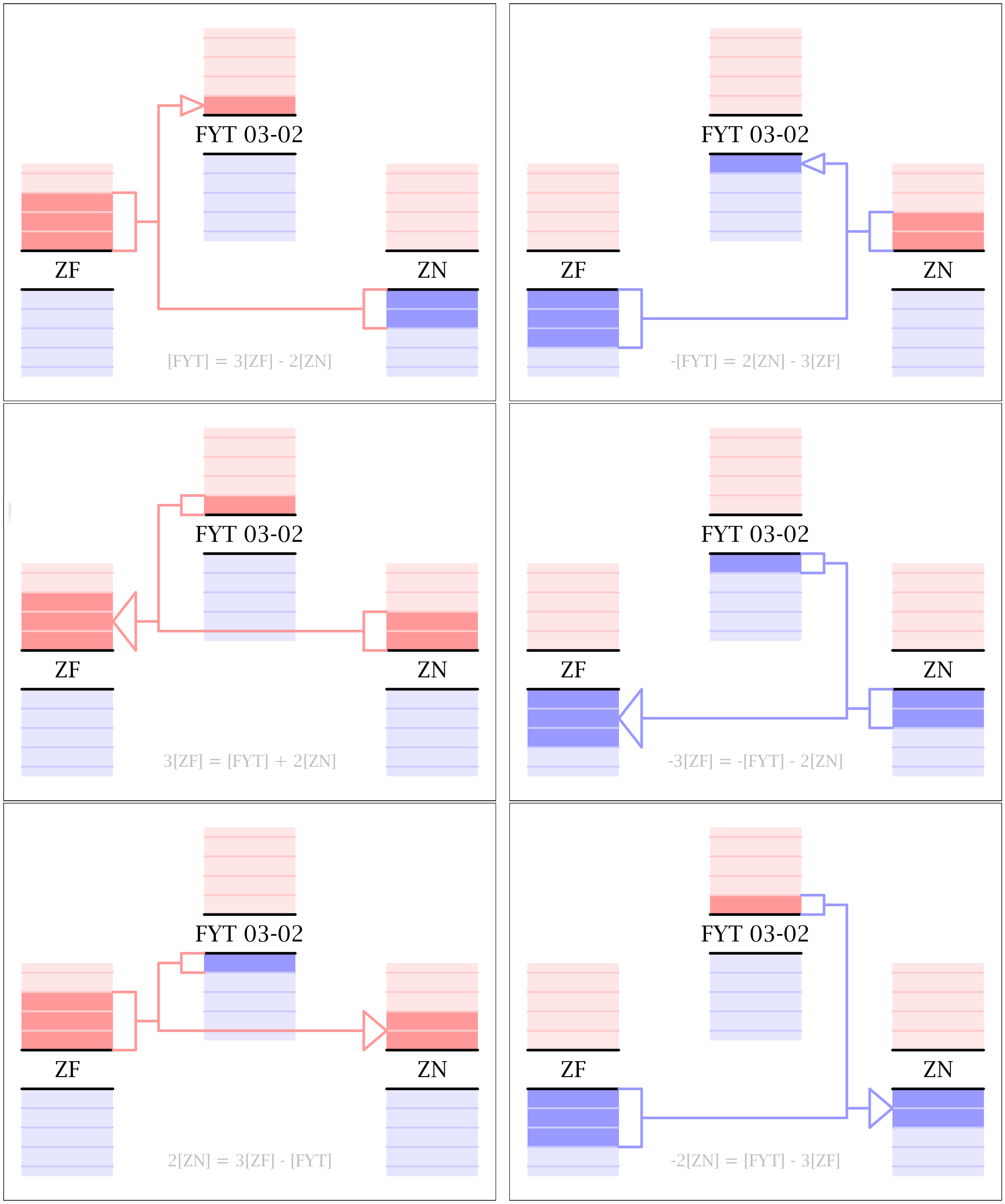
Equation (4) allows us to display bid and ask prices for the spread and for both legs in the same 2D plane. Figure 2 shows prices in the ZF contract as vertical lines, prices in ZN as horizontal lines, and prices in the FYT spread as diagonals. It also illustrates the pricing of the six possible implied trades outlined in Figure 1, shown as bolded line segments.

TABLE 1
CME outright and intercommodity Treasury spreads for March 2017 expiration. "Tick" is the minimum price increment for quotes and trades, in units of 1/32 of a point. The price ratios of spreads whose front leg is ZT are double the size ratios, since the contract size of ZT is \$200k vs. \$100k for other contracts. The tick for a spread is equal to the tick of the front leg contract. We ignore the 3-year Treasury futures because of their very low traded volume.

Instrument	Name	Tick (1/32)
ZT	2-Year Treasury Note Futures	1/4
ZF	5-Year Treasury Note Futures	1/4
ZN	10-Year Treasury Note Futures	1/2
TN	"Ultra" 10-Year Treasury Note Futures	1/2
ZB	30-Year Treasury Bond Futures	1
UB	"Ultra" Treasury Bond Futures	1

Instrument	Front Leg	Back Leg	Tick Ratio (c)	March 2017 Size Ratio	March 2017 Price Ratio ($r = p/q$)
TFY	ZT	ZF	1	1:1	2
TUF	ZT	ZF	1	4:3	2⅔
TUT	ZT	ZN	2	2:1	4
TUX	ZT	TN	2	3:1	6
TUB	ZT	ZB	4	6:1	12
TUL	ZT	UB	4	8:1	16
FYN	ZF	ZN	2	1:1	1
FYT	ZF	ZN	2	3:2	1½
FIT	ZF	ZN	2	5:3	1⅔
FIX	ZF	TN	2	5:2	2½
FOB	ZF	ZB	4	5:1	5
FOL	ZF	UB	4	6:1	6
NON	ZN	TN	1	1:1	1
TEX	ZN	TN	1	3:2	1½
NBY	ZN	ZB	2	1:1	1
NOB	ZN	ZB	2	3:1	3
NOL	ZN	UB	2	4:1	4
NCB	TN	ZB	2	2:1	2
NUB	TN	UB	2	5:2	2½
BUB	ZB	UB	1	1:1	1
BOB	ZB	UB	1	4:3	1⅓

FIGURE 1
Six modes of implication for an intercommodity spread. Each side of each of the three related markets may be implied into by one combination of the other two markets. This figure shows only the sizes of the different trades: a single lot of FYT 03-02 always matches 3 lots of ZF and 2 lots of ZN. Pricing relationships are shown in Figures 2 and 3.



The discrete grid for quotes and prices complicates this relative pricing. Let δ be the tick of the front leg contract. As noted in Table 1, the tick of the spread is δ. That means that direct quotes in the spread must be at prices that are integer multiples of δ. But implied quotes in the spread generated by relation (4) may appear on a finer grid, as we now show. Note that trades in the spread contract must always occur at multiples of δ; in "Matching and Fills" below we discuss how trades against implied quotes are generated.

Suppose that the front leg of a spread has minimum price increment δ, and the back leg has increment $c\delta$. From Table 1, we see that for the spreads and outrights of interest, c is always an integer and in fact $c \in \{1, 2, 4\}$. The settlement prices, bids, and asks of the legs must be multiples of these minimum tick sizes. Thus $A - \bar{A} = i\delta$ and $B - \bar{B} = j\,c\delta$, where i and j are arbitrary positive or negative integers. Writing $r = p/q$ where p and q have no common factors, equation (4) becomes

$$S_{\text{impl}} = i\delta - jc\delta\frac{q}{p} = \left(i - jq\frac{c}{p}\right)\delta = (ip' - jqc')\frac{\delta}{p'} \tag{5}$$

where (c', p') are (c, p) after removing common factors; that is, $p' = p/\gcd(c, p)$. Clearly, integer values of i and j give a price S_{impl} that is an integer multiple of δ/p'. Conversely, if $S_{\text{impl}} = k\,\delta/p'$, then by Bézout's identity[1] there are integers i and j so $k = ip' - jqc'$. Thus the tick size of the implied spread prices is $\delta/p' = \gcd(c, p)\delta/p$, or $1/p'$ times the tick size for direct spread prices.

For example, FYT has $c = 2$, $p = 3$ and $q = 2$, so $p' = 3$, and the tick for implied quotes is $1/3$ of the tick for direct quotes: the diagonal dashed lines in Figures 2 and 3 occur every third of a tick. NOL has $c = 2$, $p = 4$, and $q = 1$, so $p' = 2$, and the tick for implied quotes is $1/2$ of the tick for direct quotes. NCB has $c = 2$, $p = 2$, and $q = 1$, so the tick for implied quotes is the same as the tick for direct quotes.

We can repeat this pricing analysis to find the possible implied prices of the legs. Suppose a direct quote in the spread is available at price $S = k\delta$ where k is an integer. Then the relationship (4) may be used to generate implied quotes for either A or B.

- If a direct quote for B is available at price $B - \bar{B} = j\,c\delta$, then an implied quote for A is generated at price
$$A_{\text{impl}} = \bar{A} + (kp' + jqc')\frac{\delta}{p'}.$$
Since A_{impl}, like S_{impl} above, is constructed as an integer-weighted combination of two relatively prime factors p' and qc', any multiple of δ/p' can be generated as an implied price[1]. The implied tick size of A is δ/p', or $1/p'$ times the direct tick size of A.
- If a direct quote for A is available at price $A - \bar{A} = i\delta$, then an implied quote for B is generated at price
$$B_{\text{impl}} = \bar{B} + (i - k)\,\frac{p'}{qc'}\,c\delta.$$

[1] Bézout's identity: If integers a and b have no common factors, then for any k there are i, j so $ai + bj = k$.

The tick size for implied prices of B is p'/qc' times the tick size $c\delta$ for direct prices. Since qc'/p' is not necessarily an integer, the implied tick is not necessarily divisible into the direct tick. Implied quotes can be generated in the back leg only at prices that differ from the previous day's settlement price by a multiple of p'/qc' ticks.

For example, FOB has $p = 5$, $q = 1$, and $c = 4$, so $p' = 5$, $c' = 4$, and $p'/qc' = 5/4$. Implied quotes for the back leg ZB will appear on a grid whose spacing is 5/4 that of the direct prices. Many direct prices are not accessible to implied quotes and conversely.

In Figures 2 and 3, the direct spread quotes (solid diagonal lines) intersect the ZF quotes (verticle lines) to produce potential implied prices for ZN in multiples of 3/4 of a tick (3/8 of a 32nd) at 124–12+, 124–12$\frac{7}{8}$, 124–13$\frac{1}{4}$, *etc.* Intermediate prices such as 124–12$\frac{3}{4}$ cannot be produced. However, direct spread quotes can intersect with ZN quotes (horizontal lines) to produce implied prices for ZF at any multiple of 1/3 of a tick (1/12 of a 32nd). An intuitive explanation is that the points where the spread ticks intersect the ZN bid are not (in general) horizontally aligned with those where the spread ticks intersect the ZN offer (or other ZN ticks), whereas the points where the spread intersects the ZN ticks are vertically aligned, and so cover fewer total prices.

Identifying the prices of these implied quotes allows us to identify times when there are more attractive prices available than those displayed in market data. Figure 3 shows the state of the FYT market at a point in time where three such prices are available. Point (A) marks a potential trade where we could effectively sell the FYT spread at a price two thirds of a tick higher than the bid displayed on the market, point (B) shows where we could buy ZF at a price 2/3 of a tick lower than the best offer displayed, and point (C) shows where we could sell ZF at a price 3/4 of a tick higher than that displayed. Point (A) sits at the intersection of the ZF bid and the ZN offer, as the trade involves selling 3 ZF at the bid and buying 2 ZN at the offer, thus taking on a position equivalent to selling short a single FYT spread. Point (B) is at the intersection of the ZN offer and the FYT offer, as this trade involves buying both the FYT and ZN in order to go long in ZF.

MATCHING AND FILLS

In general, market orders will match against implied liquidity if there is sufficient size on all legs to trade a whole number of spreads. For price-priority purposes, implied prices are rounded outwards to the nearest tick (implied bids are rounded down and implied offers are rounded up). Direct liquidity at the rounded price has priority over implied liquidity, so the implied liquidity will be accessed only if the implied price is at least one whole tick better than the direct price, or if the market order is sufficiently large to use up all of the direct liquidity at a level.

Figure **??** shows a marketable limit order in ZF matching against implied liquidity from the FYT spread. The order first matches against the direct liquidity at the rounded price (panel B), and then matches against the implied liquidity (panel C). While there are 6 lots available implied, the order has only 5 lots left to fill after consuming the direct liquidity, and as ZF must match against the FYT in multiples of 3, only 3 of the 5 lots match against the implied liquidity. As we are using a marketable limit order, the remaining 2 lots enter the market as a limit order. The resulting market appears to be crossed, as the bid is too small to match against the implied offer. As soon as any size is added to the bid, in the form of a limit or a market order at the same price or higher, there will be sufficient quantity available to match against the implied offer, and the match will occur.

The examples shown in Figures 3 and **??** raise the question: at what prices will trades against implied liquidity print? The implied prices often do not align with the minimum price increment of the markets, and indeed often have no exact binary or decimal representation. The answer is simply that prints in market data will always be rounded outwards to the nearest minimum price increment. If someone accesses the implied liquidity at point (A) in Figure 3 by sending a large enough market order in the FYT market, then the trade will print at the direct bid in the FYT market, even as the implied legs of the trade print at the bid and offer of the ZF and ZN markets respectively. If a trader accesses the additional liquidity at point (B) by sending a large enough market order in ZF, then the trade will print at the offer in the ZF market as the implied legs print at the offers of the FYT and ZN markets.

These examples raise the additional question: what happens to the difference between the pricing described by equation (4), and the prices printed in the respective markets? For instance, at point (A) in Figure **??**, there is a 2/3 of a tick difference between the price at which a trade in the FYT will print, and the combined price at which the implied legs print. The answer is again simple but not immediately obvious: the differences will always accrue to whichever trader is acting in the spread market. In the case of point (A), the trader who issues a large enough market sell order in the FYT market will be partially filled at a price 2/3 of a tick higher than the bid price in that market, despite the fact that the published market data will print the entire trade at the bid. Similarly, in the case of point (B), when a large enough market buy is issued in the ZF market, the trader or traders whose limit orders are resting on the offer in the FYT market will actually be filled 2/3 of a tick higher than the offer price, at the price where the ZN and ZF offers intersect.

This matching behavior makes sense when we remember that the spreads do not exist as real products to be traded; they simply represent a combination of outrights. While for each position taken in the outright contracts the exchange must report that position and the exact price at which it was taken, no such restriction exists for spreads, allowing the exchange to print spread trades at rounded prices.

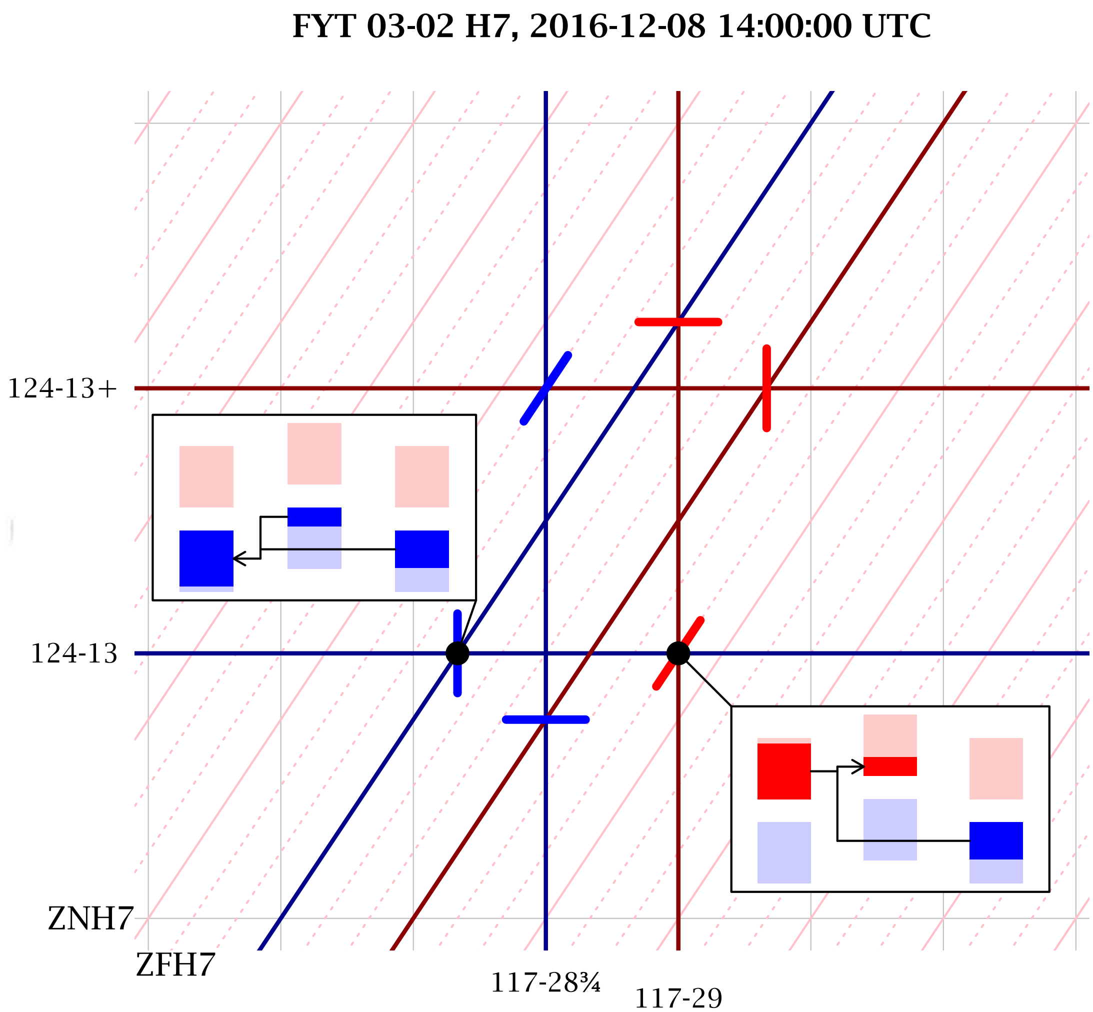


FIGURE 2. Pricing implication for the FYT 5-year vs. 10-year Treasury futures spread (March 2017 maturity, 3:2 ratio). The market is shown at a single point in time. The book for the front leg (the 5-year) is arranged along the horizontal axis, the book for the back leg (the 10-year) along the vertical axis, and the corresponding prices in the spread are marked by diagonal lines. Solid diagonal lines mark prices at which quotes on the spread can be entered directly, and dashed diagonal lines mark prices in the spread that can be obtained only implicitly through the legs. The six bolded line segments correspond to the six implied quotes diagrammed in Figure 1, with two example diagrams shown in the insets.

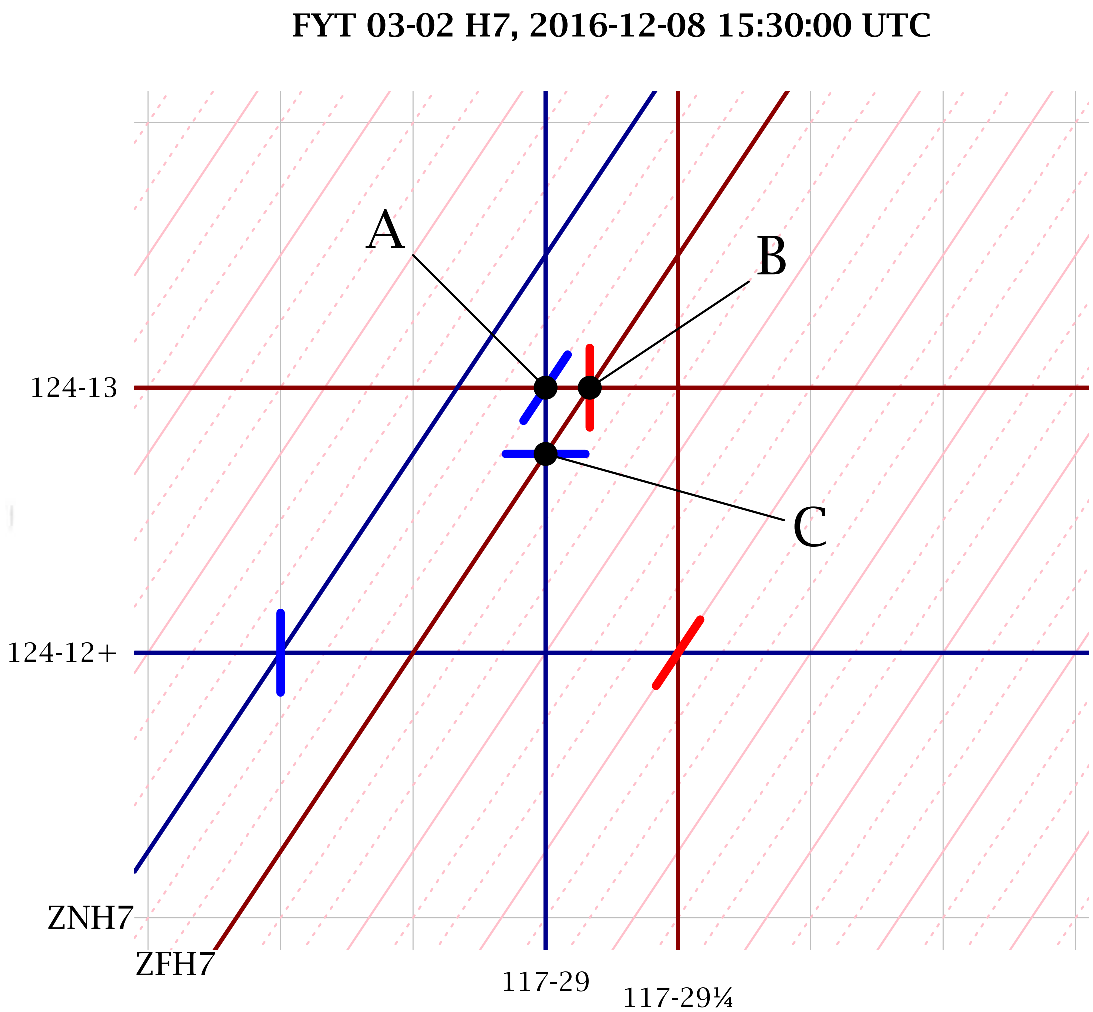


FIGURE 3. We can sometimes achieve better prices in either the spread or in one of the outrights by aggressing in the other two markets. (A) Sending market orders to buy 2 ZN and sell 3 ZF yields a better price than a market order to sell 1 FYT spread. (B) Sending market orders to buy 1 FYT spread and buy 2 ZN yields a better price than a market order to buy 3 ZF. (C) Sending market orders to buy 1 FYT spread and sell 3 ZF yields a better price than a market order to sell 2 ZN. Note that in (B) and (C), use of the spread incurs additional commission charges (as more lots change hands), whereas in (A) the commission charges are the same either way.