RISKMETRICS GROUP

RMG

CreditMetrics® Monitor

April 2008

Credit Manager User's Corner

We describe four methods to approximate the delta-gamma distribution, commonly used in Value-at-Risk calculations, and evaluate the methods for accuracy and speed. The best techniques are Partial Monte Carlo and Fourier inversion of the moment generating function. The Fourier inversion is the best unless the number of risk factors is very large (1000 – 5000 depending on the confidence level of VaR).

Risk-return reports are planned for the next version of CreditManager Currently, it is possible to extract return information, and create these reports in a separate application.

The BISTRO structure is an active synthetic Collateralized Bond Obligation. Using CreditManager, it is possible to examine worst case losses on the structure's various tranches.

CreditManager 2.0 allows for the use of multiple databases accelerating the obligor and exposure editors, and improving the organization of portfolio data.

Methods and Applications

The market upheavals of 1998 brought greater attention to market and credit risk management alike. On the credit side, last year's events pointed out that as crucial as monitoring the credit quality of counterparties is the seemingly simple task of monitoring the amounts actually exposed to these counterparties. Exposure estimation, while straightforward for traditional credit products, becomes more complex when the exposure is contingent on a market factor (e.g. an exchange rate) and, as we will see in this article, more complex still when there is a dependency between counterparty credit quality and the relevant market factor.

Vega risk can be a large part of the risk of a portfolio containing options. Any market participant owning option positions should be able to compute that risk. Vega risk is analytically easy to "nest" into the standard risk management framework. The treatment of vega risk in portfolios is, however, often impeded by the lack of availability of data on option implied volatilities.

Editor's Note

Christopher C. Finger
RiskMetrics Group
chris.finger@riskmetrics.com

With this issue, we present our first *CreditMetrics Monitor* as The RiskMetrics Group. Whle it is a first issue in a sense, it is also a last, as we will no longer be publishing our research in *CreditMetrics* or *RiskMetrics Monitors*. Our next research publication will be the inaugural issue of the *RMG Journal*. The *RMG Journal* will encompass both market and credit risk research, and continue the mix of short, practical articles with longer research pieces. While it is likely that articles on credit risk and Credit Metrics will appear in most issues, we plan to have occassional special issues devoted solely to these themes.

CreditMetrics News

Sarah Jun Xie
RiskMetrics Group
sarah.xie@riskmetrics.com

The RiskMetrics Group Spins Off from J.P. Morgan

In the fall of 1998, the RiskMetrics Group (RMG) was spun off from J.P. Morgan. J.P. Morgan and Reuters hold minority shares of RMG. RMG, known as the Risk Management Products and Research Group while at J.P. Morgan, is responsible for the creation and development of benchmark risk management products including RiskMetrics, CreditMetrics, and DataMetrics.

Planned Enhancements for CreditManager's Next Release

The next release of CreditManager, version 2.5, will be launched this summer. In this version, we plan to expand the asset type coverage, add risk/return and benchmark analysis, and continue to work on collecting and providing more credit data to our clients.

Delta-Gamma Four Ways

Jorge Mina and Andrew Ulmer
jorge.mina,andrew.ulmer@riskmetrics.com
RiskMetrics Group

We describe four methods to approximate the delta-gamma distribution, commonly used in Value-at-Risk calculations, and evaluate the methods for accuracy and speed. The best techniques are Partial Monte Carlo and Fourier inversion of the moment generating function. The Fourier inversion is the best unless the number of risk factors is very large ($1000 - 5000$ depending on the confidence level of VaR).

1 Introduction

Non-linear positions, such as options, produce portfolio returns which are frequently fat-tailed and skewed. Consequently, knowledge of the mean and variance is not enough to characterize the distribution of returns and measure Value-at-Risk (VaR). In order to calculate VaR for a non-linear portfolio, we need to obtain a percentile of the distribution of changes in portfolio value, but in general, it is impossible to obtain a closed form for the return distribution, and therefore the VaR, of a non-linear portfolio.

There are two main approaches used to calculate VaR in the non-linear case: The first involves Monte Carlo simulation to obtain a numerical estimate of VaR. This method is very accurate but can be computationally expensive for large portfolios. The second approach consists of analytical approximations of the true distribution of changes in the portfolio value. This approach can provide an approximate but fast parametric solution to the problem (e.g. , britten99). Hybrid approaches rely on delta-gamma methods to dramatically reduce the time to calculate VaR by judiciously selecting which random trials to evaluate explicitly (see).

In this article, we evaluate four different methods to obtain an analytical delta-gamma approximation of the distribution of portfolio returns using Johnson transformations (Section 3), Cornish-Fisher expansions (Section 4), Fourier methods (Section 5), and partial Monte-Carlo. Results are presented in Section 6. We conclude in Section 7.

2 The Delta-Gamma Approximation

One of the most popular methods to calculate VaR for a non-linear portfolio is the delta-gamma method, in which a second order approximation of the change in present value of the portfolio is used. The coefficients in this approximation are the first and second order sensitivities of the present value with respect to changes in the underlying risk factors. When a first order approximation is used, we refer to the method as the delta approach.

Let us assume that we have n risk factors denoted by $x = x_1, x_2, \ldots, x_n$, and that their returns $r = \frac{dx}{x}$ follow a multivariate normal distribution. Let us further assume that we have a portfolio whose value $V(x)$ is a non-linear function of the risk factors.[1]

Then, using a Taylor series expansion of $V(x)$, we can write

$$\begin{aligned} dV &= V(x+dx) - V(x) && (1) \\ &\approx \sum_{i=1}^{n} \delta_i r_i + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \Gamma_{ij} r_i r_j, && (2) \end{aligned}$$

where

$$\delta_i = x_i \frac{\partial V}{\partial x_i}, \text{ and } \Gamma_{ij} = x_i x_j \frac{\partial^2 V}{\partial x_i \partial x_j}. \qquad (3)$$

In matrix notation,

$$dV \approx \widetilde{dV} = \delta' r + \frac{1}{2} r' \Gamma r, \qquad (4)$$

where $\widetilde{dV}$ is the change in portfolio value under the delta-gamma approximation.[2]

It is important to keep in mind that our approximation is only *locally* accurate to second order as illustrated in Fig. 1. If the value of a portfolio is not a smooth and continuous function of the underlying risk factors, this approximation can provide extremely inaccurate results.

Note that the distribution of $\widetilde{dV}$ has a very general shape (Fig. 2). This property makes it useful in risk management, since it can accommodate a variety of skewed and fat tailed distributions. On the other hand, the same flexibility makes it impossible to parameterize.[3]

Now that we have a relatively simple expression for the change in the present value of our portfolio in terms of the returns on the risk factors, we have to find a way to obtain the distribution of $\widetilde{dV}$, or at least an accurate way to approximate it.

3 Johnson Transformations

Since it is impossible to obtain an analytical expression for the probability density function of $\widetilde{dV}$, the change in portfolio value under the delta-gamma approximation, we can try to fit a more tractable function by making the first n moments of the fitted and delta-gamma

[1] Risk factors can be either prices or rates. See for details.

[2] Note that these definitions differ from the standard option terminology where $\delta = \frac{\partial V}{\partial x}$ and $\Gamma = \frac{\partial^2 V}{\partial x^2}$.

[3] In the special case where $n = 1$ the distribution of $\widetilde{dV}$ is an affine transformation of a non-central chi-squared distribution.

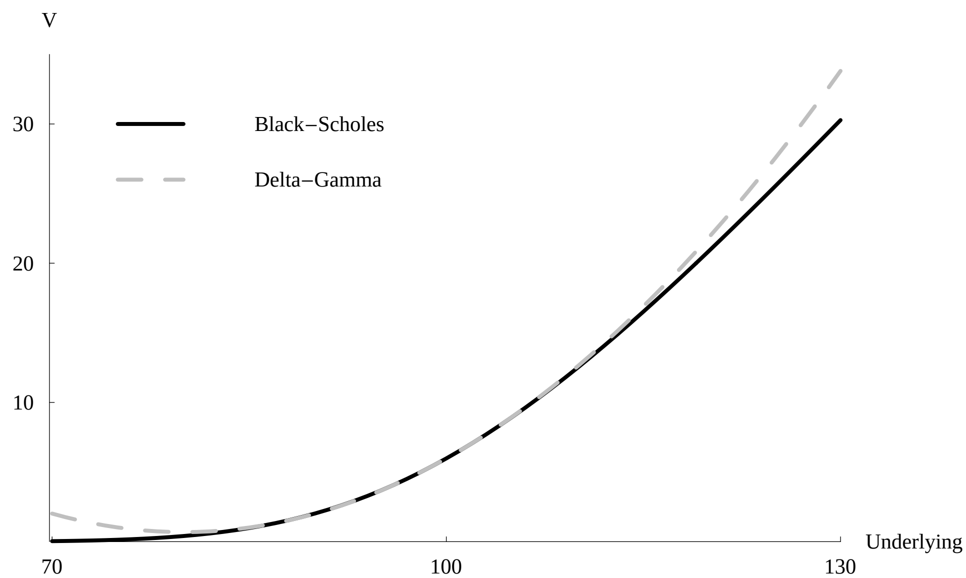


Figure 1
Call option values with Black-Scholes and delta-gamma approximation

distributions agree. It is important to understand that we are trying to approximate the distribution of $\widetilde{dV}$, which is in turn an approximation to the true distribution of dV.

We are generally not interested in the entire distribution of $\widetilde{dV}$, but only in certain percentiles of such distribution. Hence, if we can find a monotonic transformation, $f(X)$, of a random variable X, such that $f(X)$ is distributed similarly to $\widetilde{dV}$, then VaR can be approximated by:

$$\text{VaR} = f(z_\alpha), \tag{5}$$

where z_α is the α-percentile of X.[4]

Johnson (see ,) has described a set of such monotonic transformations. The transformations are:

1. Bounded on one side (lognormal)

$$f(X) = \exp\left[\frac{X-\gamma}{\delta}\right] + \xi \qquad f(X) \geq \xi, \tag{6}$$

[4] This is true because $P[f(x) \leq \text{VaR}] = P[x \leq f^{-1}(\text{VaR})]$.

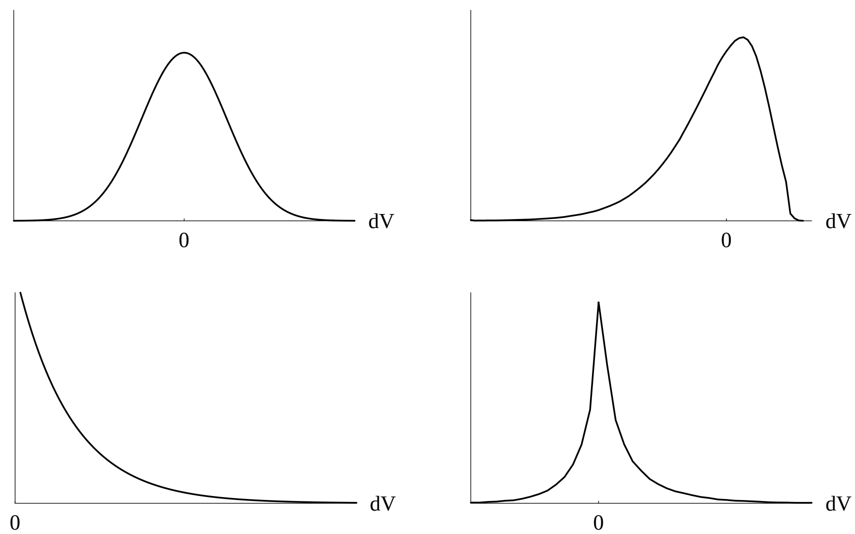


Figure 2
Examples of the diverse delta-gamma distribution, where $\widetilde{dV}$ is the change in portfolio value under the delta-gamma approximation. The diversity makes the distribution both powerful and challenging to evaluate.

2. Bounded on both sides

$$f(X) = \frac{\exp\left[\frac{X-\gamma}{\delta}\right](\xi + \lambda) + \xi}{1 + \exp\left[\frac{X-\gamma}{\delta}\right]} \qquad \xi \leq f(X) \leq \xi + \lambda, \tag{7}$$

3. Unbounded

$$f(X) = \sinh\left[\frac{X-\gamma}{\delta}\right]\lambda + \xi, \tag{8}$$

where $X \sim N(0, 1)$.

Note that these transformations depend on the four parameters γ, δ, ξ, and λ. We can fit the parameters of any of the transformations in Eqs. 6, 7, and 8 by finding the first four moments of the transformation, $f(X)$, and matching them to the moments of $\widetilde{dV}$ in Eqs. 4.[5]

[5] The use of the Johnson family of distributions to approximate the delta-gamma distribution was first described in .

Analytic expressions for the moments of the transformed random variable $f(X)$ can be found in , . The first four moments of $\widetilde{dV}$ are:

$$\mu_1 = E[\widetilde{dV}] = \frac{1}{2}tr(\Gamma\Sigma) \tag{9}$$

$$\mu_2 = E[(\widetilde{dV}-\mu_1)^2] = \delta'\Sigma\delta + \frac{1}{2}tr(\Gamma\Sigma)^2 \tag{10}$$

$$\mu_3 = E[(\widetilde{dV}-\mu_1)^3] = 3\delta'\Sigma\Gamma\Sigma\delta + tr(\Gamma\Sigma)^3 \tag{11}$$

$$\mu_4 = E[(\widetilde{dV}-\mu_1)^4] = 12\delta'\Sigma(\Gamma\Sigma)^2\delta + 3tr(\Gamma\Sigma)^4 + 3\ {\mu_2}^2 \tag{12}$$

An algorithm to fit a Johnson distribution given the first four moments can be found in . Once we fit a transformation function $f(X)$ we can obtain VaR using Eq. 5.

4 Cornish-Fisher Expansions

It is possible to obtain explicit polynomial expansions for standardized percentiles of a general distribution in terms of its standardized moments and the corresponding percentiles of the standard normal distribution.[6]

The basic result behind this kind of expansion is that if $p(x)$ is a probability density function with cumulants[7] $\kappa_1,\ \kappa_2, ...,$ then the function

$$g(x) = \sum_{i=0}^{\infty} \frac{[\sum_{j=1}^{\infty} \varepsilon_j \frac{(-D)^j}{j!}]^i}{i!} p(x), \tag{13}$$

will have cumulants $\kappa_1+\varepsilon_1,\ \kappa_2+\varepsilon_2, ...,$ where D is the differentiation operator and $D^j p(x) = d^j p(x)/dx^j$.[8]

Using Eq. 13 (see) it is possible to obtain a useful approximate representation of a distribution with known moments in terms of a known distribution $p(x)$. In the same way, it is possible to obtain polynomial expressions for the percentile of a distribution with known moments. One of these expressions is due to Cornish and Fisher (see) and can be found in .[9]

The first four terms of the Cornish-Fisher expansion for the α-percentile of $\frac{\widetilde{dV}-\mu_1}{\sqrt{\mu_2}}$ are:[10]

$$\widetilde{z_\alpha} \approx z_\alpha + \frac{1}{6}({z_\alpha}^2-1)\rho_3 + \frac{1}{24}({z_\alpha}^3-3z_\alpha)\rho_4 - \frac{1}{36}(2{z_\alpha}^3-5z_\alpha){\rho_3}^2, \tag{14}$$

[6] The application of the Cornish-Fisher expression to approximate a percentile of the delta-gamma distribution was first described in .

[7] The cumulants of a distribution are closely related to its moments and can be informally thought of as standardized moments.

[8] Note that $g(x)$ may not satisfy the condition $g(x) \geq 0$.

[9] In the Cornish-Fisher expansion, the initial distribution $p(x)$ is normal.

[10] The moments μ_i are given by Eqs. 9, 10, 11, and 12.

RMG

where

$$\rho_3 = \frac{\mu_3}{\mu_2^{\frac{3}{2}}}, \tag{15}$$

$$\rho_4 = \frac{\mu_4}{\mu_2^2} - 3, \tag{16}$$

z_α is the α-percentile of a $N(0,1)$ distribution. (17)

Therefore, we can calculate the $(1-\alpha)\%$ VaR as

$$\text{VaR} = \widetilde{z_\alpha}\sqrt{\mu_2} + \mu_1. \tag{18}$$

5 Moment Generating Functions and Fourier Transforms

The moment generating function (mgf) is another way of writing the probability density function (pdf) of the change in portfolio value, $\widetilde{dV}$. For the delta-gamma problem, we can obtain a closed-form solution for the mgf. This mgf can be inverted to yield the pdf. The inversion is performed with Fourier transforms.

The moment generating function, $M(u)$, of a random variable V is closely related to the probability density, $f(V)$

$$M(iu) = \int_{-\infty}^{\infty} e^{iuV} f(V)dV, \tag{19}$$

where i is $\sqrt{-1}$. Given the mgf, we may invert it to find the pdf with an inverse Fourier transform:

$$f(V) = \int_{-\infty}^{\infty} \frac{1}{2\pi} e^{-iuV} M(iu)du. \tag{20}$$

The moment generating function of $\widetilde{dV}$ (from) is

$$M(u) = |I - 2u\Gamma\Sigma|^{-1/2} \exp\left[\frac{1}{2}(u\Sigma\delta)'(I-2u\Sigma)^{-1}\Sigma^{-1}(u\Sigma\delta)\right] \tag{21}$$

for a $(p \times p)$ Γ matrix and $r \sim N_p(0,\Sigma)$ with Σ a positive definite covariance matrix. Equation 21 may be cast in a more computationally tractable form by replacing the determinate with eigenvalues:

$$\exp\left[\frac{u^2}{2}\sum_{j=1}^{p} b_j^2(1-2u\lambda_j)^{-1}\right]\prod_{j=1}^{p}(1-2u\lambda_j)^{-1/2}, \tag{22}$$

where P is an orthogonal matrix such that $P'\Sigma^{1/2}\Gamma\Sigma^{1/2}P = \text{diag}(\lambda_j,\ j=1,p)$, where the λ's are the eigenvalues of $\Sigma^{1/2}\Gamma\Sigma^{1/2}$, and $b = P'(\Sigma^{1/2}\delta)$.

With the moment generating function, the pdf is calculated using Eq. 20, and then integrated to determine the cumulative density function, and therefore VaR. The inversion (Eq. 20) is evaluated numerically using a Fast Fourier Transform (FFT) which performs the integral in order[11] $N \ln N$, where N is the (evenly spaced) number of points at which the pdf of the change in value is calculated. Using FFT's to numerically approximate Fourier integrals is discussed in . The integral can be evaluated to relatively high accuracy with only $N = 2^8$ points equally spaced in an integration range of $\pm N\sigma^2/10$, where σ^2 is the variance of the delta-gamma distribution is given by Eq. 10. For higher accuracy, N and the factor of 10 in the integration range may both be increased.

Because of symmetries in the Fourier integral, the moment generating function needs to be evaluated at only $N/2$ points. Since N is so small, the only computational bottleneck is in finding the eigenvalues of $\Sigma^{1/2}\Gamma\Sigma^{1/2}$. This calculation takes order M^3 operations where M is the number of risk factors and takes about 3 minutes on an 450 MHz NT desktop for 500 risk factors.

6 Numerical Results and Discussion

We evaluate the delta-gamma methods by using them to calculate VaR on four test portfolios. The portfolios are:

1. A three month short put on the S&P 500 three standard deviations out of the money.
2. A three month at the money long put on the S&P 500.
3. A Yen bear spread composed of a three month short put on Yen one and a half standard deviations out of the money and a three month long put on Yen one standard deviation out of the money.
4. A delta-hedged Yen put composed of a three month short put on Yen at the money and a cash hedge equivalent to the Yen-delta of the put.

The pdf's for $\widetilde{dV}$ for each portfolio are shown in Fig. 3.[12] We see that the delta-gamma approximation captures the expected features of the true distribution for these portfolios. For example, in the first portfolio we are short a deep out of the money put, so we expect to make a small profit with a large probability (note the spike), and lose a large amount of money with a small probability if the S&P 500 drops by a large amount. Similarly, in portfolio 4 we see that once we delta-hedged our short put position we remain short gamma causing the distribution to have a heavy left tail.

[11] An operation is which is order N, takes no more than cN basic operations, where c is a constant. Examples of basic operations are addition, multiplication, and exponentiation.

[12] The figures were generated using the output from a 2^{12} point FFT.

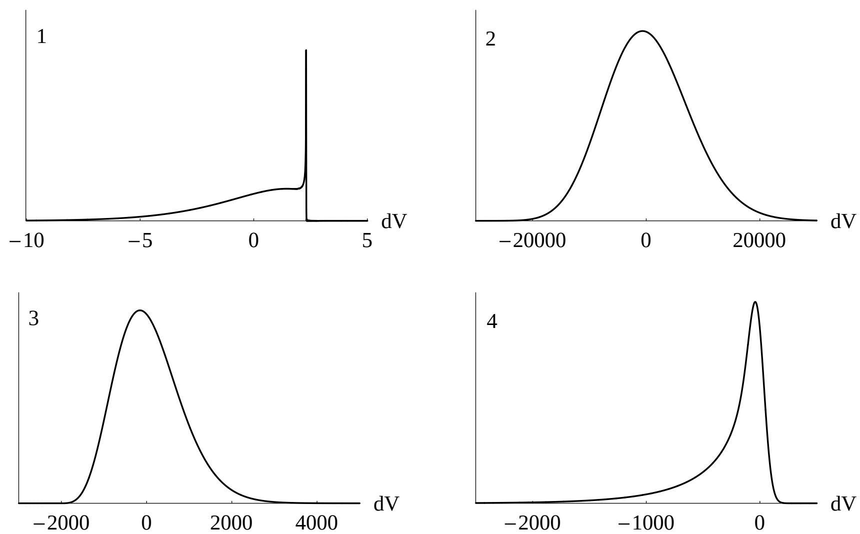


Figure 3
Test portfolios probability distributions of the change in portfolio value under the delta-gamma approximation, $\widetilde{dV}$.

The percentiles of $\widetilde{dV}$ as measured by the delta-gamma approximations are shown in table 1. The full Monte Carlo and delta methods are shown for comparison with the delta-gamma approximation models.[13]

The Partial Monte Carlo is a Monte Carlo simulation of the delta-gamma distribution that is performed by generating deviates with a multivariate normal distribution and then valuing them using the simple delta-gamma formula, Eq. 4. The Partial Monte Carlo and FFT are exact in that they converge to the distribution of $\widetilde{dV}$. The FFT differs slightly from the Partial Monte Carlo because the FFT was run with only 128 evaluations of the moment generating function, while Partial Monte Carlo was run with 10000 simulations.

The only method discussed in the paper that does not appear in the table is the Johnson method for which it was difficult to fit a distribution for some of the portfolios.[14] While the portfolios were chosen to stress the methods, they are not improbable positions to be held within a portfolio. We conclude that Johnson is not a robust choice for performing delta-gamma.

[13] Note that with the full Monte-Carlo method we obtain the percentiles of the true distribution of changes in the portfolio, dV.
[14] The reason for this is the limited set of shapes attainable with the Johnson transformations.

Table 1
Delta-gamma Percentiles

		1%	5%	10%	90%	95%	99%
Portfolio 1	Monte Carlo	-9.08	-5.46	-3.94	2.11	2.5	3.07
	Partial Monte Carlo	-7.74	-4.96	-3.61	1.97	2.20	2.30
	FFT	-8.06	-5.11	-3.73	1.95	2.18	2.46
	Cornish-Fisher	-8.04	-5.09	-3.72	1.99	2.26	2.41
	Delta	-5.17	-3.66	-2.85	2.85	3.66	5.17
Portfolio 2	Monte Carlo	-15282	-11308	-9099	10297	13408	19578
	Partial Monte Carlo	-15581	-11474	-8923	10041	13247	19171
	FFT	-15508	-11350	-8983	10043	13102	19041
	Cornish-Fisher	-15482	-11310	-8963	10028	13065	18992
	Delta	-17237	-12187	-9496	9496	12187	17237
Portfolio 3	Monte Carlo	-1589	-1200	-958	1219	1605	2434
	Partial Monte Carlo	-1387	-1050	-868	1092	1469	2202
	FFT	-1380	-1058	-859	1111	1471	2202
	Cornish-Fisher	-1374	-1056	-858	1108	1467	2195
	Delta	-1784	-1261	-983	983	1261	1784
Portfolio 4	Monte Carlo	-1849	-1061	-757	22.00	51.00	102.00
	Partial Monte Carlo	-1896	-1063	-777	22.26	49.07	102.23
	FFT	-1859	-1083	-766	27.12	57.92	117.06
	Cornish-Fisher	-1978	-1109	-765	-34.09	-81.28	-205.78
	Delta	-142.64	-100.85	-78.58	78.58	100.85	142.64

The Cornish-Fisher expansion gives very good results for all portfolios except portfolio 4, where it gives poor and nonsensical answers, i.e. the first percentile is less than the fifth. This occurs because the Cornish-Fisher is a polynomial approximation for the percentiles, and in certain circumstances, it has a maximum in the range of percentiles. Because the Cornish-Fisher is an extremely fast algorithm, we would recommend it for a quick look at the delta-gamma values but caution that it may give unacceptable results in extreme circumstances.

The FFT and Partial Monte Carlo are competitive in that they are both robust and give relatively fast answers. When compared with the delta approximation and the full Monte Carlo examples, it is clear that while the delta-gamma method does not give exact answers, it gives a good approximation and is a marked improvement over the delta method.

The speed of the delta-gamma methods is their main attraction. The RiskMetrics delta method requires between one to four evaluations (for numerical derivatives) of an entire portfolio for typical portfolios in order to determine the deltas, an evaluation of the covariance matrix, and then an order N^2 matrix multiplication on the deltas and the covariance matrix, where N is the number of factors. The delta-gamma approximations all require only a few more portfolio evaluations than delta in order to find gamma. The Cornish-Fisher approximation requires almost no additional computation.

The FFT algorithm requires an order N^3 matrix operation, and a couple hundred evaluations of the moment generating function which requires order N calculations. For large portfolios with many risk factors, the N^3 matrix operation is the slowest (which means the marginal cost of making the FFT approximation very accurate is small) and requires roughly 3 minutes $\cdot\, (N/500)^3$ on a 450 MHz NT desktop.

The Partial Monte Carlo is slower than the FFT unless the number of factors is very large, because the Partial Monte Carlo requires order $N_{\mathrm{sim}} N^2$ operations where N_{sim} is the number of simulations required which depends on the confidence level, but is ~ 1000 for 95% VaR and $\sim 5,000$ for 99% VaR. The main advantage of Partial Monte Carlo over the FFT is the Partial Monte Carlo's relative ease of implementation.

As a final speed comparison, we note that the Monte Carlo takes as long as Partial Monte Carlo, plus the time to evaluate all of the positions N_{sim} times which will be the dominate contribution when the portfolio contains many complex instruments that that are based on many factors.

7 Conclusions

We conclude that even for the extreme portfolios that we considered, the delta-gamma approximation is very close to the full Monte Carlo simulations and offers a significant improvement over the delta approximation. The delta-gamma methods are two-tiered with regard to speed. The Johnson and Cornish-Fisher methods are fast, but less accurate and occasionally unstable. We do not recommend these methods except for fast implementations or for quick checks of a more accurate algorithm.

The best techniques are Partial Monte Carlo and Fourier inversion of the moment generating function. The Fourier inversion is the best choice for speed and accuracy unless the number of risk factors is very large ($1000 - 5000$ depending on the confidence level of VaR). These implementations are not significantly more complex than the RiskMetrics delta method which is now extremely popular, they can still be computed much faster than full Monte Carlo simulation, and they are integral to hybrid approaches to full Monte Carlo.

References

Britten-Jones, M. & Schaefer, S. M. (1999) Non-linear Value-at-Risk, European Finance Review, Vol. 2, Num. 2, 1-27.

Cardenas, J. et al.(1999) Monte Carlo within a Day, Risk, Vol. 12, Num. 2, 55-59.

Cornish, E.A., and Fisher, R.A. (1937). Moments and cumulants in the specification of distributions, Review of the International Statistical Institute, 5, 307-320.

Johnson, N.L. (1949). Systems of frequency curves generated by methods of translation, Biometrika, 36, 149-176.

Johnson, N.L., Samuel Kotz, and N. Balakrishnan (1994). *Continuous Univariate Distributions*, Volume 1, John Wiley & Sons.

Hill, I.D., Hill, R., & Holder, R. L. (1976), Fitting Johnson Curves by Moments (Algorithm AS 99), Applied Statistics, 180-189.

Mathai, A. M. & Provost, S. B., (1992) *Quadratic Forms in Random Variables, Theory and Applications*, Marcel Dekker, Inc., New York, Chapter 3.

Morgan Guaranty Trust Company (1996). RiskMetrics Technical Document, 4th ed.

Press, W. H., Teukolsky, S. A., Vetterling, W. T., & Flannery, B. P. (1996). *Numerical Recipes in C*, Cambridge University Press, Chapter 13.

Zangari, P. (1996). A VaR methodology for portfolios that include options, RiskMetrics Monitor, 1st quarter, 4-12

Risk-return Reporting

Christopher C. Finger
chris.finger@riskmetrics.com
RiskMetrics Group

Risk-return reports are planned for the next version of CreditManager Currently, it is possible to extract return information, and create these reports in a separate application.

One of the most common and most natural requests for future releases of the Credit Manager product is for the facility to compare the current risk outputs with an expected return measure. While this is planned for future releases, it is possible to perform some risk-return analysis in the current version. This article describes how a user might obtain return information using the current software to complement the existing risk outputs.[1]

1 The Default Database

CreditManager 2.0 uses a flat file Paradox database which contains records of all market data, reports, obligors, and exposures. After installation is complete, the default database will be located in the PC file system in a folder in the CreditManager 2.0 directory called **Data2**.[2]

1.1 Creating additional databases

The easiest way to create a new database is to copy an existing database folder, such as the default Data2 folder. The following screen shot shows the file system in which resides the new, duplicated database, NewDataBase.

Portfolio Distribution

To compute the MGF for our portfolio distribution, we again rely on the conditional independence of the individual obligors. Letting V_1, $V_2 \ldots V_N$ denote the values of the individual loans, and E_Z the expectation conditional on Z, we have that

$$E_Z e^{t \cdot V} = E_Z \exp\left[t \cdot (V_1 + \ldots + V_N)\right] = E_Z \left(e^{t \cdot V_1} \cdot \ldots \cdot e^{t \cdot V_n}\right) \quad . \tag{1}$$

Now since the V_i are conditionally independent and have the same stand alone distributions, we may write the right-hand side of Equation 11 as the product of individual moment generating functions.

---

[1] This article is partly based upon discussions with John Veidis at Fuji Bank, New York.
[2] In this case we uses the Paradox database to access detailed information of each of the following: market data, reports, obligors, and exposures.

Chart 1
Here is a chart caption.

Chart here

Figure 1
Here is a figure caption.
Exposures grouped by current value.

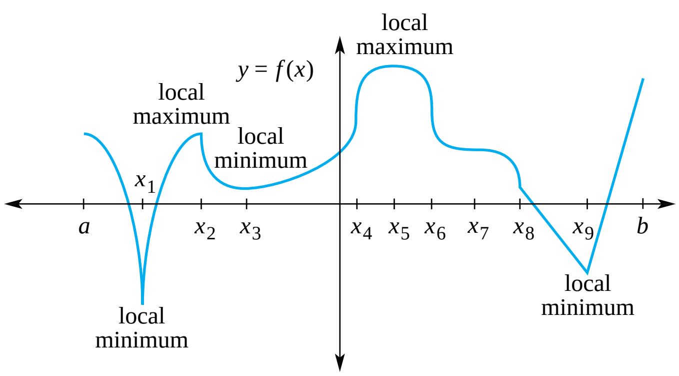


Figure 2
How to Place Temporary Figure Box for .eps File

`figsamp.eps`

Table 1
Effects of Government Ownership of Capital (only labor is taxed)

Fraction of capital owned	0	2.5	5	10
Average tax rate	0.1059	0.1054	0.1049	0.1041
Standard deviation of tax rate	0.0074	0.0082	0.0089	0.0105
Average capital stock	0.1059	0.1061	0.1063	0.1066
Standard deviation of capital stock	0.0139	0.0141	0.0143	0.0147

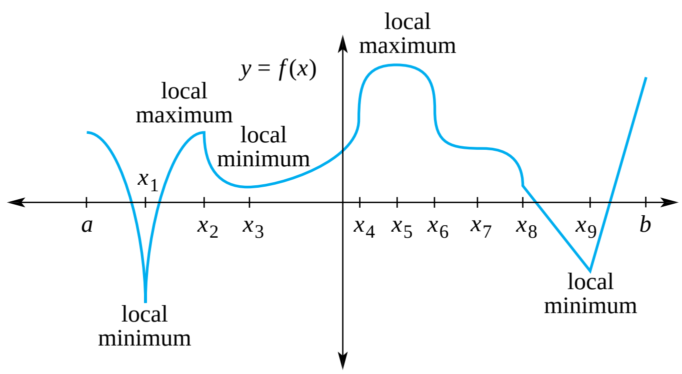
Chart 2
Here is a chart caption.

Chart here

Here is nofloatchart:

Here is nofloatfigure:

Figure 3
Here is a figure caption.
Exposures grouped by current value.



Here is nofloattable:

Table 2
Effects of Government Ownership of Capital
(only labor is taxed)

Fraction of capital owned	0	2.5	5	10
Average tax rate	0.1059	0.1054	0.1049	0.1041
Standard deviation of tax rate	0.0074	0.0082	0.0089	0.0105
Average capital stock	0.1059	0.1061	0.1063	0.1066
Standard deviation of capital stock	0.0139	0.0141	0.0143	0.0147

Extensions of the basic case

The three strong assumptions we used in the previous section were:

- only one market factor drove all of the asset value processes,
- there were only two possible credit states, and
- all of our exposures were identical—that is, exposiure had the same size default probability, recovery rate, and weight on the market factor.

In this section, we will first present the framework and notation for a more general case, then discuss how our three earlier methods can be extended into a MonteCarlo setting, and finally present resultas fro an example portfolio.

REMARK 1: The number of scenarios to be considered does not depend on the number of exposures. If there were 10 exposures instead of the assumed 5, the only difference would be that in the PGF's, the exponent term would be changed from 5 to 10.

Lemma 1 (The General Dcomposition): *Suppose there exist:*

1. *An integer* $M > 0$.
2. *Real numbers* $p_{ij} \in [o, 1]$ *for* $i = 1, \ldots, M$ *and* $j = 1, \ldots, N$ *and*
3. *Real numbers* $\lambda_i \in [0, 1]$ *for* $i = 1, \ldots M$

PROBLEM 1: *(Sufficient conditions for existence of decompositions)* Given *a priori* data on default rates and correlations does there exist a solution to the decomposition problem described in the Lemma?

PROBLEM 2: *(Parameterization of admissible decompositions)* Assuming admissible solutions to the decomposition problem described in the Lemma exist, can we obtain an easily parameterized family of solutions?

ASSUMPTION 1: The default correlations are such that the matrix of correlations is positive semidefinite.

Theorem 1 (Sufficient Conditions for Decomposition & Extreme Decomposition I): *Let Assumptions 1 and 2 hold, then...*

Conclusion

We have disucssed a number of possibilities for improving upon the standard Monte Carlo approach used currently in CreditMetrics. All of the possibilites are based on the observation that given the moves in the market factors that drive our obligors, the individual obligor credit moves are conditionally independant.

References

Amemiya, Takeshi. *Advanced Econometrics.* Cambridge, Mass.: Harvard University Press, 1985.

Andreyenkov, Vladimir G., Vasily D. Patrushev, and John P. Robinson. *Rhythm of Everyday Life: How Soviet and American Citizens use Time.* Boulder, Colo.: Westview Press, 1989.

Bailey, Martin J. "The Welfare Cost of Inflationary Finance," *Journal of Political Economy,* vol. 64 (April 1956), pp. 93-110.

Ball, Laurence. "Credible Disinflation with Staggered Price-Setting," *American Economic Review,* vol. 84 (March 1994a), pp. 282-89.

_________. "What Determines the Sacrifice Ratio," in N. Gregory Mankiw, ed., *Monetary Policy.* Chicago: The University of Chicago Press, 1994b.

Barro, Robert J., and Chaipat Sahasakul. "Measuring the Average Marginal Tax Rate from the Individual Income Tax," *Journal of Business,* vol. 56 (October 1983), pp. 419-52.

Baumol, William J. "The Transactions Demand for Cash: An Inventory-Theoretic Approach," *Quarterly Journal of Economics,* vol. 66 (November 1952), pp. 545-66.

Benabou, Roland. "Inflation and Markups: Theories and Evidence From the Retail Trade Sector," *European Economic Review,* vol. 36 (April 1992), pp. 566-74.

Bureau of the Census. *Historical Statistics of the United States, Colonial Times to 1970.* Washington: U.S. Department of Commerce, 1975.

Chadha, Jagjit S., Andrew G. Haldane, and Norbert G. J. Janssen. "Shoe-Leather Costs Reconsidered." Manuscript. Bank of England, January 1997.

Chari, V.V., Lawrence J. Christiano, and Patrick J. Kehoe. "Optimality of the Friedman Rule in Economies with Distorting Taxes," *Journal of Monetary Economics*, vol. 37 (April 1996), pp. 203-23.

Correia, Isabel, and Pedro Teles. "The Optimal Inflation Tax," Discussion Paper 123. Institute for Empirical Macroeconomics, Federal Reserve Bank of Minneapolis, August 1997.

Dotsey, Michael, and Peter N. Ireland. "The Welfare Cost of Inflation in General Equilibrium," *Journal of Monetary Economics*, vol. 37 (February 1996), pp. 29-47.

Federal Reserve Bank of Kansas City. *Achieving Price Stability.* Kansas City: Federal Reserve Bank of Kansas City, 1996.

Worst Loss Analysis of BISTRO Reference Portfolio

Toru Tanaka, VP
Risk Management
Fuji Bank

Sheikh Pancham PhD, AVP
Risk Management
Fuji Bank
spanchum@bloomberg.net

Tamunoye Alazigha, AVP
Risk Monitoring and Control
Fuji Bank

The BISTRO structure is an active synthetic Collateralized Bond Obligation. Using Credit-Manager, it is possible to examine worst case losses on the structure's various tranches.

Here is some text.

Summary

And in summary, we can determine...

Appendix

Proof of the Lemma

Here we show that if there exist λ and p_{ij} with the attributes defined in the Lemma, then the M scenarios, viewed as M mutually exclusive outcomes, produce the desired default rates.

$$p_{jmax} \geq p_j - \alpha_s u_{sj} \geq P_{jmin} \tag{A.1}$$

Using Multiple Databases

Rob Fraser
RiskMetrics Group Europe
rob.fraser@riskmetrics.com

CreditManager 2.0 allows for the use of multiple databases accelerating the obligor and exposure editors, and improving the organization of portfolio data.

Here is some text.

Citations include Abramsky *et al.* (1994), Nerode and Shore (1997), and Gentzen (1935).

References

Abramsky, S., Gabbay, D., and Maibaum, T., editors (1994). *Handbook of Logic in Computer Science III — Semantic Structures*. Oxford University Press.

Gentzen, G. (1935). Untersuchungen über das logische Schließen. *Mathematische Zeitschrift*, **39**, 176–210 & 405–431.

Nerode, A. and Shore, R. (1997). *Logic for Applications*. Springer-Verlag, 2 edition.

Toward a better estimation of wrong-way credit exposure

Christopher C. Finger
chris.finger@riskmetrics.com
RiskMetrics Group

The market upheavals of 1998 brought greater attention to market and credit risk management alike. On the credit side, last year's events pointed out that as crucial as monitoring the credit quality of counterparties is the seemingly simple task of monitoring the amounts actually exposed to these counterparties. Exposure estimation, while straightforward for traditional credit products, becomes more complex when the exposure is contingent on a market factor (e.g. an exchange rate) and, as we will see in this article, more complex still when there is a dependency between counterparty credit quality and the relevant market factor.

1 Introduction

The market upheavals of 1998 brought greater attention to market and credit risk management alike. On the credit side, last year's events pointed out that as crucial as monitoring the credit quality of counterparties is the seemingly simple task of monitoring the amounts actually exposed to these counterparties. Exposure estimation, while straightforward for traditional credit products, becomes more complex when the exposure is contingent on a market factor (e.g. an exchange rate) and, as we will see in this article, more complex still when there is a dependency between counterparty credit quality and the relevant market factor.

Regulators have explicitly recognized the uncertain future credit exposure on swaps, forwards, and other derivative contracts. The Basle Capital Accord requires regulatory capital for current exposure – roughly, the amount which would be lost should the counterparty default today – plus additional "add-on" capital to account for the potential future exposure – the cost of replacing a contract some time in the future – due to moves in the underlying market factor. As to estimating and monitoring exposure, sophistication among practioners has varied greatly. To address these discrepancies, twelve large commercial and investment banks formed the Counterparty Risk Management Policy Group (CRMPG) in January, 1999, and produced a report in June, 1999. Fifth of the group's twelve recommendations was that financial intermediaries "should upgrade their ability to monitor and, as appropriate, set limits for various exposure measures".

The CRMPG report also highlights four issues that complicate the analysis of credit exposure. The issues read like a laundry list of risk management themes in general. Liquidity, event, and operational concerns are the first three issues. The fourth is the typical assumption that the credit quality of the counterparty is independent of the market factors that underlie the exposure to the counterparty. The report proposes stress tests to evaluate the impact of relaxing this assumption. We will show in this article that the independence assumption is actually utilized in the applications of exposure measures, and that by examining these applications, it is possible to extend exposure measures in a natural way. The extension allows us to begin with any assumption about the distribution of the underlying risk factor, and to account for dependency between credit quality and market moves without resorting to stress tests.

The remainder of this article is structured as follows: in the Section 2, we define a number of standard measures of credit exposure and discuss their applications; in Section 3, we develop a framework to extend these measures by considering the dependency between counterparty credit quality and the underlying market factor; in Section 4, we present an example exposure calculation using this framework; in Section 5, we discuss a technique to calibrate the parameters of the model; lastly, we summarize and conclude.

2 Definitions and uses of exposure measures

In this section, we define a number of exposure measures and discuss their applications. For further details on the definitions and calculations, see Zangari (1997a) and (1997b).

The first distinction between measures is whether they are estimates of current or potential exposures. The definition of *current exposure*, if not its calculation, is straightforward: the current exposure of a contract is the cost of replacing the contract, should the counterparty default today. Intuitively, this is just the current mark to market value of the contract, if the value is positive. If the mark to market value is negative, the current exposure is zero, since the counterparty has no obligations, and there is no cost to replace the contract. In practice, as the CRMPG report points out, in illiquid markets or for large positions, the replacement cost of a contract will likely be greater than its mark to market value. We will not address this issue here, and will assume that at any time, the mark to market value of a contract gives an accurate assessment of its replacement cost.

In this article, we will treat measures of *potential exposure*. For these measures, we are concerned with the consequences of a counterparty default some time in the future. Thus, we would like to estimate the replacement cost of the contract, given that the counterparty defaults at some future date. If we fix the date, then we may treat the value (and replacement cost) of the contract on that date as a random variable, and define two basic exposure measures. The *expected exposure* is the expected replacement cost of the contract in the case of a counterparty default; the *maximum exposure* is the worst replacement cost, given some level of confidence, that we might incur should the counterparty default.

To make these definitions more concrete, and to aid our discussion later, we introduce some notation. For simplicity, we will assume that only one risk factor underlies our contract. Suppose we wish to estimate exposure at some future date t. Let R_t denote the (random) value of the risk factor at time t, and f_t denote the probability density function[1] for R_t. Let $v_t(r)$ be the mark to market value of the contract, and $E_t(r) = \max\{0, v_t(r)\}$ be the exposure, at time t given that the risk factor at that time is equal to r. The expected exposure at t is

$$\mathbf{E}[E_t(R_t)] = \int_0^\infty dr\ f_t(r) \cdot E_t(r). \tag{1}$$

The maximum exposure at t, at confidence level q, is the level x satisfying

$$q = \mathbf{P}\{E_t(R_t) < x\} = \int_{\{r:E_t(r)<x\}} dr\ f_t(r). \tag{2}$$

Since expected and maximum exposure refer to a specific date, there are actually entire profiles of these measures over the life of the contract. The profile of expected exposure, for example, consists of the measure defined in (1) for every t between today and the maturity of the contract. For practical reasons, it is common to aggregate these profiles into one number. For the expected exposure profile, the aggregate measure is referred to as *average exposure*, and is defined as the weighted average of the expected exposure measures, with weights proportional to the discount factors for each t. The aggregate measure for the maximum exposure profile is referred to as *peak exposure* and is defined as the maximum value over the entire maximum exposure profile. We will not concern ourselves further with these aggregate measures, but point out that the methods introduced below can easily be applied in the aggregate, as well as in the single horizon case.

Generically, all exposure measures are utilized to facilitate comparisons between traditional credit products (where the exposure is a fixed quantity) and contracts where the exposure is contingent on one or more underlying market factors. For this reason, it is common to see any of the exposure measures defined above referred to as *loan equivalent exposures*. The basic idea is that all of the effects of market volatility are embedded in the exposure measure, so for credit risk management purposes, a contract with a loan equivalent exposure of 100 can be treated like a loan with a face value of 100.

That the basic premise of loan equivalence is not exactly true is evident in that banks typically use different exposure measures for different purposes. The CRMPG report[2] mentions the emerging practice of credit charges, whereby the credit risk on a swap or derivative contract is transferred within the institution, and the valuation of the contract is adjusted via an internal charge for the "cost of credit". In a sense, this practice can be thought of as the business that originates the contract buying credit protection from another part of the organization. The cost of this protection, as well as the pricing impacts it may have on the contract itself, are most often based on notions of expected default loss, and therefore use expected or average exposure measures. On the other

[1] In this article, we will assume f_t is known. For details on modeling risk factors for exposure estimation, see Zangari (1997a) or Jamshidian and Zhu (1997).

[2] Section II D, "Valuation and Exposure Management", pages 29-31.

hand, firm-wide counterparty exposure limits are designed to bound the worst case loss in the case of default. Thus, it is typical to apply maximum or peak exposure measures against these limits.

Vega risk

Allan M. Malz
RiskMetrics Group
allan.malz@riskmetrics.com

Vega risk can be a large part of the risk of a portfolio containing options. Any market participant owning option positions should be able to compute that risk. Vega risk is analytically easy to "nest" into the standard risk management framework. The treatment of vega risk in portfolios is, however, often impeded by the lack of availability of data on option implied volatilities.

1 Introduction

Vega risk can be a large part of the risk of a portfolio containing options. Any market participant owning option positions should be able to compute that risk. Vega risk is analytically easy to "nest" into the standard risk management framework. The treatment of vega risk in portfolios is, however, often impeded by the lack of availability of data on option implied volatilities. Vega risk is also complicated by the prevalence of volatility smiles and term structures in most option markets. Volatility smiles, in spite of their occasionally treacherous effects on option books, are often neglected by risk managers. This paper provides a guide to incorporating vega risk into a "classical" value-at-risk (VaR) model. The paper includes a tractable approach to capturing the effects of the volatility smile and term structure on vega risk and their interaction with other risk factors. In our discussion, we will present several examples using a high-quality database of foreign exchange implied volatilities.[1]

1.1 Definition of vega risk

Option positions are exposed to a range of market risks. **Delta** and **gamma** risk are the exposures of an option position to changes in the prices of the underlying assets. **Vega** is the exposure of an option position to changes in the implied volatility of the option:

$$\text{vega} = \frac{\partial \text{option value}}{\partial \text{implied volatility}}.$$

The change in the option value is defined as a partial derivative, that is, it assumes all other factors determining the option value, such as the current level of the underlying asset price and the remaining time to maturity, are held constant. Vega is measured in

[1] The source of the database is the J.P. Morgan foreign exchange desk. The database includes implied volatilities for a wide range of currency pairs and maturities, and includes extensive coverage of volatility smiles and is available from the RiskMetrics Group.

dollars or other base currency units. Implied volatility is generally measured in percent per annum. Units of implied volatility are often called **vols**, so dollar-yen might have an implied volatility of 12 percent or "12 vols". In this document, we will always express implied volatility as a decimal, so one vol equals 0.01.[2]

References

Abramsky, S., Gabbay, D., and Maibaum, T., editors (1994). *Handbook of Logic in Computer Science III — Semantic Structures*. Oxford University Press.

Gentzen, G. (1935). Untersuchungen über das logische Schließen. *Mathematische Zeitschrift*, **39**, 176–210 & 405–431.

Nerode, A. and Shore, R. (1997). *Logic for Applications*. Springer-Verlag, 2 edition.

[2] The terminology can be a bit confusing, since an implied volatility itself may also be referred to as a "vol," as in "equity index vols have been rising of late."

Worldwide CreditMetrics Contacts

RiskMetrics Group

Americas
Sarah Jun Xie (1-212) 981-7424
sara_xie@riskmetrics.com

Europe/Asia
Rob Fraser (44-171) 842-0262
rob.fraser@riskmetrics.com

Co-sponsors

Arthur Andersen
Jitendra Sharma (1-212) 708-4536
jitendra.d.sharma@arthurandersen.com

Bank of America
David E. Gibbs (1-415) 953-1352
david.e.gibbs@bankamerica.com

Arthur Andersen
Jitendra Sharma (1-212) 708-4536
jitendra.d.sharma@arthurandersen.com

Bank of America
David E. Gibbs (1-415) 953-1352
david.e.gibbs@bankamerica.com

Deutsch Bank
James Glover (61-2)9258-2411
james.glover@db.com

Arthur Andersen
Jitendra Sharma (1-212) 708-4536
jitendra.d.sharma@arthurandersen.com

Bank of America
David E. Gibbs (1-415) 953-1352
david.e.gibbs@bankamerica.com

Arthur Andersen
Jitendra Sharma (1-212) 708-4536
jitendra.d.sharma@arthurandersen.com

Bank of America
David E. Gibbs (1-415) 953-1352
david.e.gibbs@bankamerica.com

Deutsch Bank
James Glover (61-2)9258-2411
james.glover@db.com

Arthur Andersen
Jitendra Sharma (1-212) 708-4536
jitendra.d.sharma@arthurandersen.com

Bank of America
David E. Gibbs (1-415) 953-1352
david.e.gibbs@bankamerica.com

Arthur Andersen
Jitendra Sharma (1-212) 708-4536
jitendra.d.sharma@arthurandersen.com

Bank of America
David E. Gibbs (1-415) 953-1352
david.e.gibbs@bankamerica.com

Deutsch Bank
James Glover (61-2)9258-2411
james.glover@db.com

Arthur Andersen
Jitendra Sharma (1-212) 708-4536
jitendra.d.sharma@arthurandersen.com

Bank of America
David E. Gibbs (1-415) 953-1352
david.e.gibbs@bankamerica.com

Arthur Andersen
Jitendra Sharma (1-212) 708-4536
jitendra.d.sharma@arthurandersen.com

Bank of America
David E. Gibbs (1-415) 953-1352
david.e.gibbs@bankamerica.com

Deutsch Bank
James Glover (61-2)9258-2411
james.glover@db.com

Arthur Andersen
Jitendra Sharma (1-212) 708-4536
jitendra.d.sharma@arthurandersen.com

Bank of America
David E. Gibbs (1-415) 953-1352
david.e.gibbs@bankamerica.com

Arthur Andersen
Jitendra Sharma (1-212) 708-4536
jitendra.d.sharma@arthurandersen.com

Bank of America
David E. Gibbs (1-415) 953-1352
david.e.gibbs@bankamerica.com

Deutsch Bank
James Glover (61-2)9258-2411
james.glover@db.com

Arthur Andersen
Jitendra Sharma (1-212) 708-4536
jitendra.d.sharma@arthurandersen.com

Bank of America
David E. Gibbs (1-415) 953-1352
david.e.gibbs@bankamerica.com

Arthur Andersen
Jitendra Sharma (1-212) 708-4536
jitendra.d.sharma@arthurandersen.com

Bank of America
David E. Gibbs (1-415) 953-1352
david.e.gibbs@bankamerica.com

Deutsch Bank
James Glover (61-2)9258-2411
james.glover@db.com

Arthur Andersen
Jitendra Sharma (1-212) 708-4536
jitendra.d.sharma@arthurandersen.com

Bank of America
David E. Gibbs (1-415) 953-1352
david.e.gibbs@bankamerica.com

Arthur Andersen
Jitendra Sharma (1-212) 708-4536
jitendra.d.sharma@arthurandersen.com

Bank of America
David E. Gibbs (1-415) 953-1352
david.e.gibbs@bankamerica.com

Deutsch Bank
James Glover (61-2)9258-2411
james.glover@db.com

CreditMetrics Products

Introduction to CreditMetrics™: A broad overview of the CreditMetrics methodology and practical applications.

CreditMetrics™—Technical Document: A comprehensive reference on the CreditMetrics methodology. The document begins with an overview and simple examples. Later chapters include details on parameter estimation, the model's assumptions, and siumlation framework.

CreditMetrics®Monitor: A semiannual publication which discusses a variety of credit risk management issues, ranging from practical implementations to modeling and statistical questions.

CreditMetrics data sets: Current market data (foreign exchange rates, yield curves, and spread curves by industry and rating category), as well as derived data (industry correlations and transition matrices). Current market data and industry correlations are updated weekly.

All of the above can be downloaded from the Internet at *www.creditmetrics.com*.